■ Load Packages

Model Solver: Load me first Read me last

Car as Depreciating Resellable Asset with Maintenance

It is widely acknowledged that driving an automobile off the new-car dealer's lot immediately decreases the value of that automobile by several thousand dollars. There are several reasons for this spread, among them the "Lemons" problem famously described by George Akerlof. After assuming ownership of the car, you gain a large amount about the car's condition, maintenance history, involvement in wrecks and predilection for breakdowns -- much more so than any potential buyer. Given asymmetric information between the buyer and seller and multiple price points corresponding to condition, a buyer must assume that a car has "average" condition (as she has no reason to expect it is higher or lower). Thus no rational seller would place a high-quality car on the market, as he would receive below fair value for his (known) high-quality car. In turn buyers, must then assume "below-average" price, and in the absence of means to reliably determine quality, there will be eventually only "lemons" (bad-quality) cars on the market. Specifically, even a one-year used car -- lacking as it does the assurance of dealer reputation, and carrying the risk that the seller knows the car is inferior -- has some rational basis for trading at a large discount to an otherwise equivalent new car.

In practice, institutions such as dealerships, mechanic pre-sale checkups, fair-practice laws, seller's reputation, and CarFax help to signal quality and preserve the market for a used car. The market for used cars is liquid and widely available, and used cars have prices that are well-defined: entities such as the NADA (National Automobile Dealer's Association), the Kelley Blue Book and Edmunds.com list prices for used cars by year and condition.

■ The Value of a Car

At a basic level, you own a car for one primary purpose -- carrying you and your passengers around safely, quickly and reliably -- and you would like to do so at minimum total cost. Each car's value, then, consists in large part of the total forward expectation of all costs associated with buying the car, maintaining it, and replacing it when it dies or becomes economically sound to do so. Since all these actions cost money and since the market value of the car decreases with time, the "value" of a car is in fact a large negative number.

A perfectly economically rational creature will choose an optimal control strategy for buying and replace her car, one that maximizes its total forward value (that is, a strategy that minimizes the total costs incurred). Using the optimal control algorithms we've learned in class and publicly-available market data, we can enumerate exactly what this strategy entails.

Define

V_a:	value of car of age *a*
$C_{\text{buy},a}$, $C_{\text{sell},a}$:	market price for an *a*-year old car, with cost to buy possibly different than cost to sell.
$\delta = \frac{1}{1+r\,\Delta t}$:	Discount factor (capturing the opportunity cost of having an asset in the future)
c_a:	costs incurred by keeping the car for the coming year
$f_{a,k}$:	Reward function for a policy decision *k* in state *a*: for instance, the costs incurred by choosing to keep your 10-year-old car or those incurred by selling it and buying a new one.
$T_{ab,k}$:	Decision function: probability of going from state *a* to state *b* under policy *k*.

Let's first set up the simplest interesting model. There is only one kind of car at all in the world: the Honda Accord-4 Cyl. Sedan Four-Door LX. Each year, you will decide either to keep your *a*-year-old Honda (incurring maintenance on an age *a* car and giving you an $a+1$-aged car a year from now) or to sell it and buy the most recent used Honda (incurring maintenance on the age-1 car, giving you an age 2 car next term, and gaining the resale value of your old car). The value of a car will then satisfy the Bellman equation

$$V_a = \text{Max}\left\{ \begin{matrix} -h_a + & \delta\ V_{a+1} \\ -h_1 - C_{\text{buy},1} + C_{\text{sell},a} & +\,\delta\, V_2 \end{matrix} \right\}$$

and we can use recursive iteration to find both V_a and the optimal control strategy.

▪ Market Price

We considered two models of car, the Honda Accord-4 Cyl. Sedan Four-Door LX and the Ford Crown Victoria V8 Four-Door Sedan to be the only car in our universe. As both models have very similar and well-behaved prices, our initial analysis will only examine the Honda.

For both cars, using Edmunds.com, we assembled data for the current market price of the Ford and the Honda Accord from model years 1990 - 2006. In each case, we chose the exactly equivalent model and specified the base configuration for that model year by hitting "submit" on the form without adding or subtracting options. We spot-checked the resulting model to ensure that there were no sharp discontinuities in model tiers or optional components.

Given our assumption that all cars of a certain model decay in value at the same rate, we fit this data to construct an estimate for the sale and purchase price for the given car model as a function of age. For the sale price, we used the average of the "Trade-In" and "Private Party" prices, assuming that people had an equally likely future expectation of selling to a private party as trading the car in at the dealer. Similarly, we used an average of the "Private Party" and "Dealer Retail" prices for the purchase price. Both prices were found to agree reasonably well to an exponentially depreciating fit with a constant offset ($C = \alpha + C_o\, e^{-\beta a}$ for set α, β, C_o):

$$C_{\text{sell,honda}}(a) = 19102\, e^{-0.132\, a} - 1626 \quad C_{\text{buy,honda}}(a) = 19698\, e^{-0.127\, a} - 1091$$
$$C_{\text{sell,ford}}(a) = 16911\, e^{-0.209\, a} - 312 \quad C_{\text{buy,ford}}(a) = 18138\, e^{-0.195\, a} - 83$$

▪ Car maintenance

We found a survey of household automotive expenditures (in Stern, Hartmann and Engers 2005) that showed the cost of car maintenance could be roughly described as following the function over the regime 0, 10 years:

$$h_a = 229.7 + 18.739\, a$$

This curve is, somewhat surprisingly, linear. Why is that? One factor is, according to Stern, that as a car increases in age it will often be driven less and relied on less often -- you take the new car on a trip to the beach, and you might even pass it along to your son or keep it as a second car. Another factor is that this function shows repair *expenditures*, not repairs incurred -- there is a selection bias against large repair costs for cars with little residual value. (If your mechanic says your $1200 Honda requires $5000 worth of engine it is likely you should scrap the car and put that money towards a new vehicle). Later we will experiment with adding a quadratic term on an ad-hoc basis, but for now we have no reason to do anything other than extrapolate from the best indicator we have.

■ Asset Maintenance

■ A Look at the symbolic Form

```
In[80]:= (*carvalueModel[aMax,δ,repl,Cbuy,Csell,Ctrans,Cmaint,Pdead]*)
         With[{aMax = 6},
           {δxs, fis, Pxs, replJunk, keepJunk} =
            carvalueModel[aMax, δ, {1}, C_{sell,#} &, C_{sell,#} &, 0, C_{maint,#} &, 0 &];
           Vis = Table[V_i, {i, 1, aMax + 1}];
          ];
         vcSym = Vchoices[Vis, {δxs, fis, Pxs}];
         TableForm /@ (Most /@ vcSym)
```

Out[82]=

$$\left\{\begin{matrix} \delta V_2 - C_{\text{maint},1} \\ \delta V_2 - C_{\text{maint},1} - C_{\text{sell},1} + C_{\text{sell},2} \\ \delta V_2 - C_{\text{maint},1} - C_{\text{sell},1} + C_{\text{sell},3} \\ \delta V_2 - C_{\text{maint},1} - C_{\text{sell},1} + C_{\text{sell},4} \\ \delta V_2 - C_{\text{maint},1} - C_{\text{sell},1} + C_{\text{sell},5} \\ \delta V_2 - C_{\text{maint},1} - C_{\text{sell},1} + C_{\text{sell},6} \end{matrix}, \begin{matrix} \delta V_2 - C_{\text{maint},1} \\ \delta V_3 - C_{\text{maint},2} \\ \delta V_4 - C_{\text{maint},3} \\ \delta V_5 - C_{\text{maint},4} \\ \delta V_6 - C_{\text{maint},5} \\ -\infty \end{matrix}\right\}$$

For this unrealistically short allowable lifetime ($a_{\text{max}} = 6$), the first column shows the costs incurred with the "replace" policy; the second the costs for the "keep" policy. The reward at max age is set to -∞ to ensure replacement; in practice, we can specify a maximum age that lies outside of any prudently attainable car age.

■ Solve

```
In[83]:= (*carvalueModel[aMax,δ,repl,Cbuy,Csell,Ctrans,Cmaint,Pdead]*)
Module[{aMax = 100, nIterations = 100, δ = 1/(1 + 0.03), repl},
  repl = {1};
  {δx1, fi1, Px1, replJunk, keepJunk} =
   carvalueModel[aMax, δ, repl,
    accordSalePrice, accordSalePrice, 0,
    BasicMaintenanceCosts, 0 &];
  ViSolvedAll1 =
   NestList[Vnext[#, {δx1, fi1, Px1}] &, Array[0 &, aMax + 1], nIterations];
  ViSolved1 = Last[ViSolvedAll1];
  Vnext1 = Vchoices[ViSolved1, {δx1, fi1, Px1}];
  policies1 = Append[repl, "keep"];
 ];

prettyTable[{"Value"}, Range[Length[ViSolved1]], {ViSolved1}]
prettyTable[policies1, Range[Length[ViSolved1]], Vnext1]
prettyTable[{"Best", "Second Best"},
 Range[Length[ViSolved1]], policy[policies1, Vnext1, 2]]
```

Out[84]=

	1	2	3	4	5	6	7	8	9	10
Value	−22759.4	−23250.8	−23738.5	−24221.7	−24701.1	−25176.6	−25648.	−26115.3	−26578.2	−27035.

Out[85]=

	1	2	3	4	5	6	7	8	9	10
1	−22822.	−24890.1	−26702.7	−28291.4	−29683.9	−30904.4	−31974.1	−32911.7	−33733.5	−34453.8
keep	−22822.	−23314.2	−23802.1	−24286.3	−24766.7	−25243.1	−25715.5	−26183.7	−26646.7	−27105.3

Out[86]=

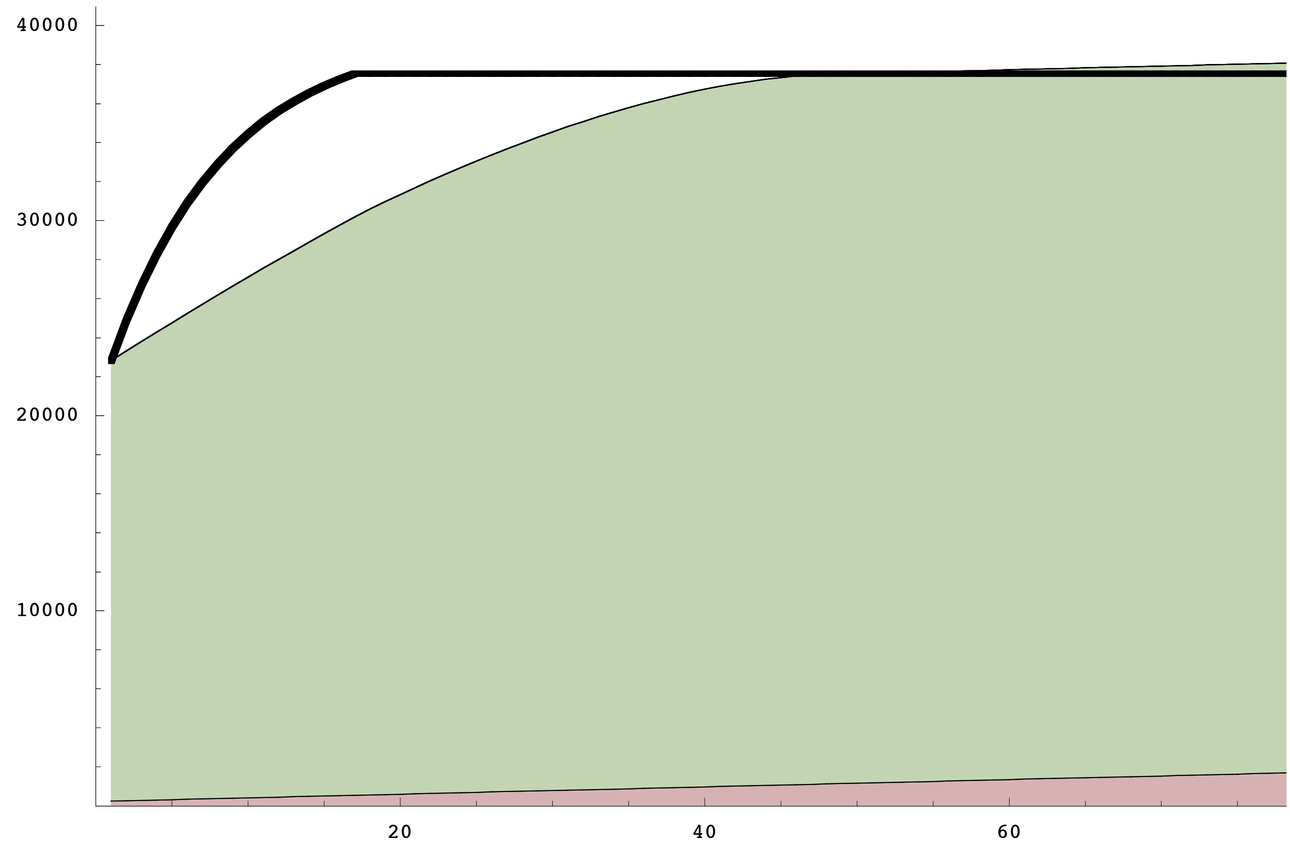
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Best	keep	keep	keep	keep	keep	keep	keep	keep	keep	keep	keep	keep	keep	keep	keep	keep
Second Best	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

```
In[87]:= replNoResale1 = Plus @@ (replJunk /. {V_i_ :> ViSolved1[[i]]});
(*DisplayTogether[{
   StackedPlot[replJunk/.{V_i_:>ViSolved1[[i]]}],
   ListPlot[replNoResale1]
  }];*)
replBestYear1 = Ordering[replNoResale1, 1] // First
replNoResale1[[replBestYear1]]
```

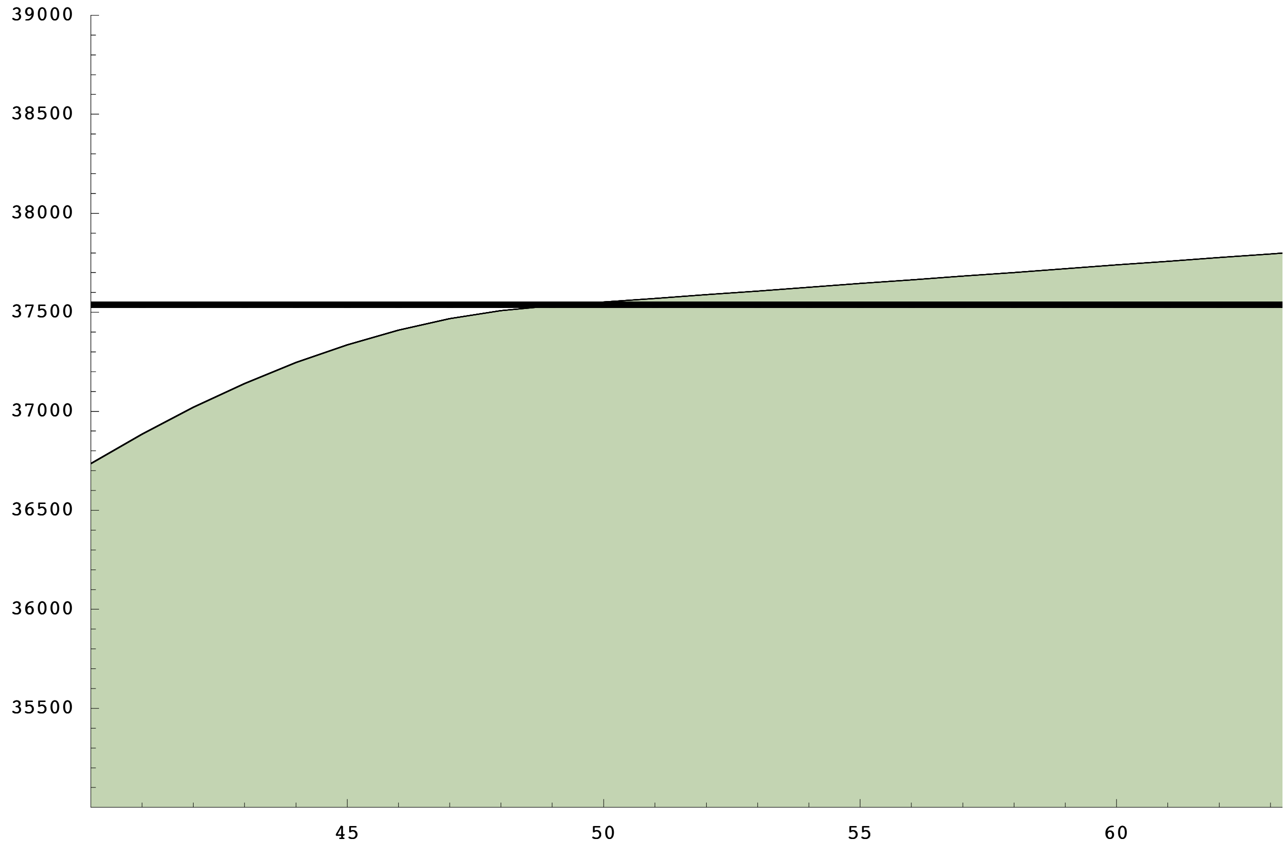
Out[88]= 1

Out[89]= 37939.1

```
In[90]:= DisplayTogether[{
   StackedPlot[keepJunk /. {Vi_ :> ViSolved1[[i]]}],
   ListPlot[Table[replNoResale1[[replBestYear1]] - accordSalePrice[i], {i, 1,
      Last[Dimensions[keepJunk]]}], PlotJoined -> True, PlotStyle -> Thickness[0.005]]
  }, PlotRange -> {{0, 85}, {0, Automatic}} ];
```



```
In[91]:= DisplayTogether[{
           StackedPlot[keepJunk /. {Vi_ :→ ViSolved1[[i]]}],
           ListPlot[Table[replNoResale1[[replBestYear1]] - accordSalePrice[i], {i, 1,
              Last[Dimensions[keepJunk]]}], PlotJoined → True, PlotStyle → Thickness[0.005]]
          }, PlotRange → {{40, 65}, {35000, 39000}}];
```



■ Explanation of results

With this simple model we see the cheery fact that a car is worth negative several thousand dollars -- from just past -$20,000 at 1 year to some -$35,000 past age 50 or so. The optimal strategy is to hold on to your car until age 50, at which point the modestly increasing repair costs will mitigate to getting a a brand-new car.

Now obviously in the real world people choose a *much* smaller time horizon for vehicle replacement. What's going on?

▪ We're Gonna Need a Bigger Boat

Here are several factors to add to the model.
. Cars die (needs a new engine, wrecked): value→0, repurchase forced.
. A person can buy any year of car, not just age-1.
. There is a spread between purchase and resale.
. Repair catastrophes can lead to lost wages or the opportunity cost of a ruined vacation, time waiting for tow truck, etc.
. Transaction costs buying/selling a car, plus opportunity costs of the hassle shopping/hawking the car.

Junkyards: Car as Asset with Replacement and Chance of Scrappage

▪ Scrappage

We were able to find a research survey (on website of S Stern, University of Virginia) that extracted values for the rate of "Car Death" as a function of the age of the car. As expected, in the first few years the rate of complete failure was very low, around 2%. The curve is concave up and reaches approximately 12% for 10 year old cars, rising above 20% by year 13. Using this we added another option for cars as they aged, allowing them to age normally as well as "die". Dying meant that the car's selling price was reduced to zero and that it had to be replaced. Accomplishing this required adding another state for the car and altering the transition matrix to evolve the car into the dead state with a probability that was extracted from the measured scrappage rate curve.

In real life, cars that die from catastrophic mechanical failure are, as likely to have residual value as to actually cost money for haulage to the junk yard. (This is slightly unfair to a newer car but, we think, a second-order effect). Owners of cars that are totalled in accidents get an insurance claim from their old car. However, our model does not include insurance premiums in the cost of a car; if those premiums are priced rationally, the expected loss of value due to dead-car-from-accident and the cost+benefits of insurance should wash. Therefore, we can fairly assign a value of zero to a dead car.

■ Solve

In[132]:=
```
(*carvalueModel[aMax,δ,repl,Cbuy,Csell,Ctrans,Cmaint,Pdead]*)
Module[{aMax = 90, nIterations = 100, δ = 1/(1 + 0.03), repl},
  repl = {1};
  {δx2, fi2, Px2, replJunk, keepJunk} = carvalueModel[aMax, δ, repl,
    accordSalePrice, accordSalePrice, 0,
    BasicMaintenanceCosts,
    deathRateHonda];
  ViSolvedAll2 = NestList[Vnext[#, {δx2, fi2, Px2}] &, Array[0 &, aMax + 1], nIterations];
  ViSolved2 = Last[ViSolvedAll2];
  Vnext2 = Vchoices[ViSolved2, {δx2, fi2, Px2}];
  policies2 = Append[repl, "keep"];
 ];

prettyTable[{"Value"}, Range[Length[ViSolved2]], {ViSolved2}]
prettyTable[policies2, Range[Length[ViSolved2]], Vnext2]
prettyTable[{"Best", "Second Best"},
 Range[Length[ViSolved2]], policy[policies2, Vnext2, 2]]
```

Out[133]=

	1	2	3	4	5	6	7	8	9	10
Value	−51615.4	−52771.5	−53962.1	−55188.7	−56375.5	−57503.5	−58560.3	−59538.9	−60436.6	−61253.9

Out[134]=

	1	2	3	4	5	6	7	8	9	10
1	−51728.	−53796.1	−55608.7	−57197.4	−58589.9	−59810.4	−60880.1	−61817.7	−62639.5	−63359.8
keep	−51728.	−52884.3	−54075.1	−55302.	−56489.1	−57617.3	−58674.3	−59653.1	−60551.	−61368.4

Out[135]=

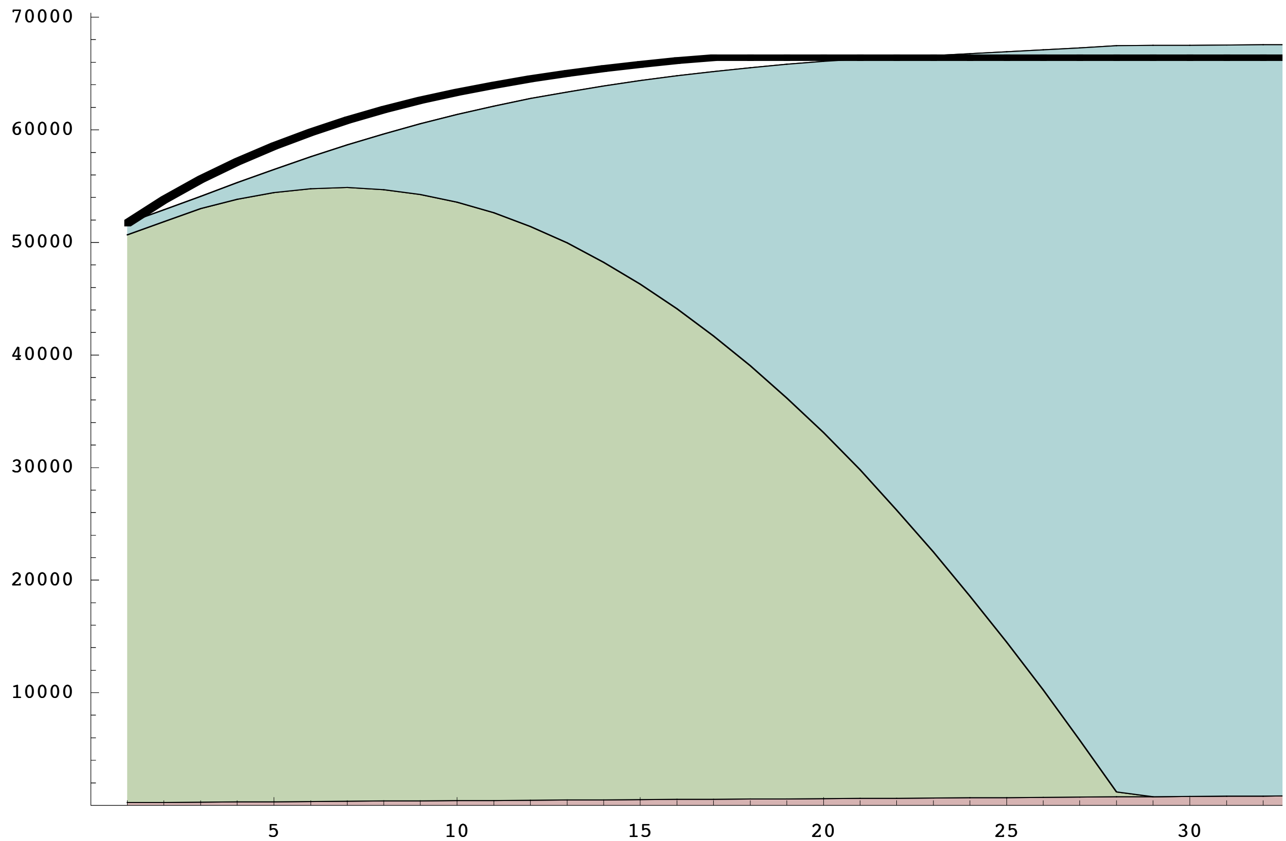
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Best	keep	keep	keep	keep	keep	keep	keep	keep	keep	keep	keep	keep	keep	keep	keep	keep	keep
Second Best	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

In[93]:=
```
replNoResale2 = Plus @@ (replJunk /. {V_i_ :> ViSolved2[[i]]});
replBestYear2 = Ordering[replNoResale2, 1] // First
replNoResale2[[replBestYear2]]
```

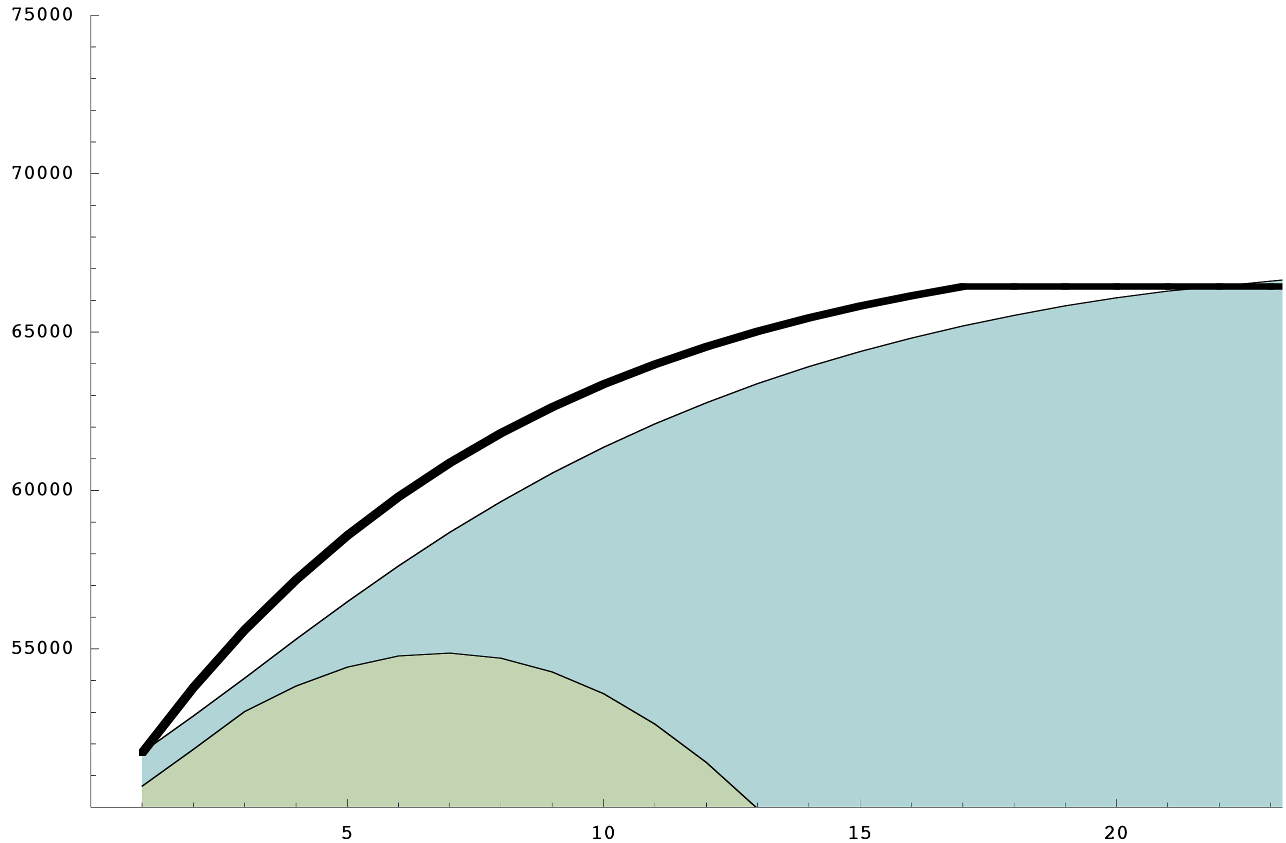
Out[94]= 1

Out[95]= 66845.1

```
In[96]:= DisplayTogether[{
           StackedPlot[keepJunk /. {Vi_ :→ ViSolved2[[i]]}],
           ListPlot[Table[replNoResale2[[replBestYear2]] - accordSalePrice[i], {i, 1,
              Last[Dimensions[keepJunk]]}], PlotJoined → True, PlotStyle → Thickness[0.005]]
          }, PlotRange → {{0, 35}, {0, Automatic}} ];
```



```
In[97]:= DisplayTogether[{
           StackedPlot[keepJunk /. {Vi_ :> ViSolved2[[i]]}],
           ListPlot[Table[replNoResale2[[replBestYear2]] - accordSalePrice[i], {i, 1,
              Last[Dimensions[keepJunk]]}], PlotJoined → True, PlotStyle → Thickness[0.005]]
         }, PlotRange → {{0, 25}, {50000, 75000}}];
```



Opportunity Costs

■ Lost Wages

We were able to find data on the average maintenance costs associated with owning a car for a year, but the slow linear growth of this cost with respect to the age of the car was insufficient to describe actual purchasing behavior. We concluded that simple maintenance, like oil changes or tune-ups, did not fully capture the costs associated with an unreliable car and that it was necessary to add a term that described the opportunity costs incurred because of vehicle malfunctions. In order to get a rough estimate for these costs, we assumed that the mechanical failure rate as a function of age should look similar to the scrappage rate described above. Additionally, we speculated that 5% of mechanical failures would lead to scrappage, so that the frequency of mechanical failures could be drawn simply by scaling the scrappage rate curve by a factor of 20. Finally, we found that the average US wage is around $36,000 and assuming that the typical breakdown results in the loss of 1/3 of a day, the cost associated with each malfunction is about $50.

■ Transaction Cost

When we allowed for the decision to buy a replacement car of any age it was discovered that the optimal policy was to sell a newer car and replace it with sginificantly older versions, a behavior that is clearly not frequently expressed in reality. To further explore this we decided to add a transaction cost associated with buying and selling an automobile. The amount we chose was equal to 2 days worth of the average US salary or about $300 -- this is the opportunity cost associated with the hassle of buying or selling a new car: visiting dealerships, test drives, checking out Craigslist-advertised cars, taking out a classified ad, cleaning the car for resale.

■ Solve

In[136]:=

```
(*carvalueModel[aMax,δ,repl,Cbuy,Csell,Ctrans,Cmaint,Pdead]*)
Module[{aMax = 90, nIterations = 100, δ = 1/(1 + 0.03), repl},
  repl = {1};
  {δx2, fi2, Px2, replJunk, keepJunk} = carvalueModel[aMax, δ, repl,
    accordPurchasePrice, accordSalePrice, 2 * Vdaily,
    BasicMaintenanceCosts,
    deathRateHonda];
  ViSolvedAll2 = NestList[Vnext[#, {δx2, fi2, Px2}] &, Array[0 &, aMax + 1], nIterations];
  ViSolved2 = Last[ViSolvedAll2];
  Vnext2 = Vchoices[ViSolved2, {δx2, fi2, Px2}];
  policies2 = Append[repl, "keep"];
 ];

prettyTable[{"Value"}, Range[Length[ViSolved2]], {ViSolved2}]
prettyTable[policies2, Range[Length[ViSolved2]], Vnext2]
prettyTable[{"Best", "Second Best"},
 Range[Length[ViSolved2]], policy[policies2, Vnext2, 2]]
```

Out[137]=

	1	2	3	4	5	6	7	8	9	10
Value	−55353.	−56606.3	−57898.6	−59231.7	−60522.6	−61750.4	−62901.4	−63967.9	−64946.7	−65838.3

Out[138]=

	1	2	3	4	5	6	7	8	9	10
1	−56907.4	−58975.4	−60788.	−62376.8	−63769.3	−64989.7	−66059.5	−66997.1	−67818.9	−68539.2
keep	−55474.	−56727.6	−58020.1	−59353.5	−60644.6	−61872.7	−63023.9	−64090.6	−65069.6	−65961.4

Out[139]=

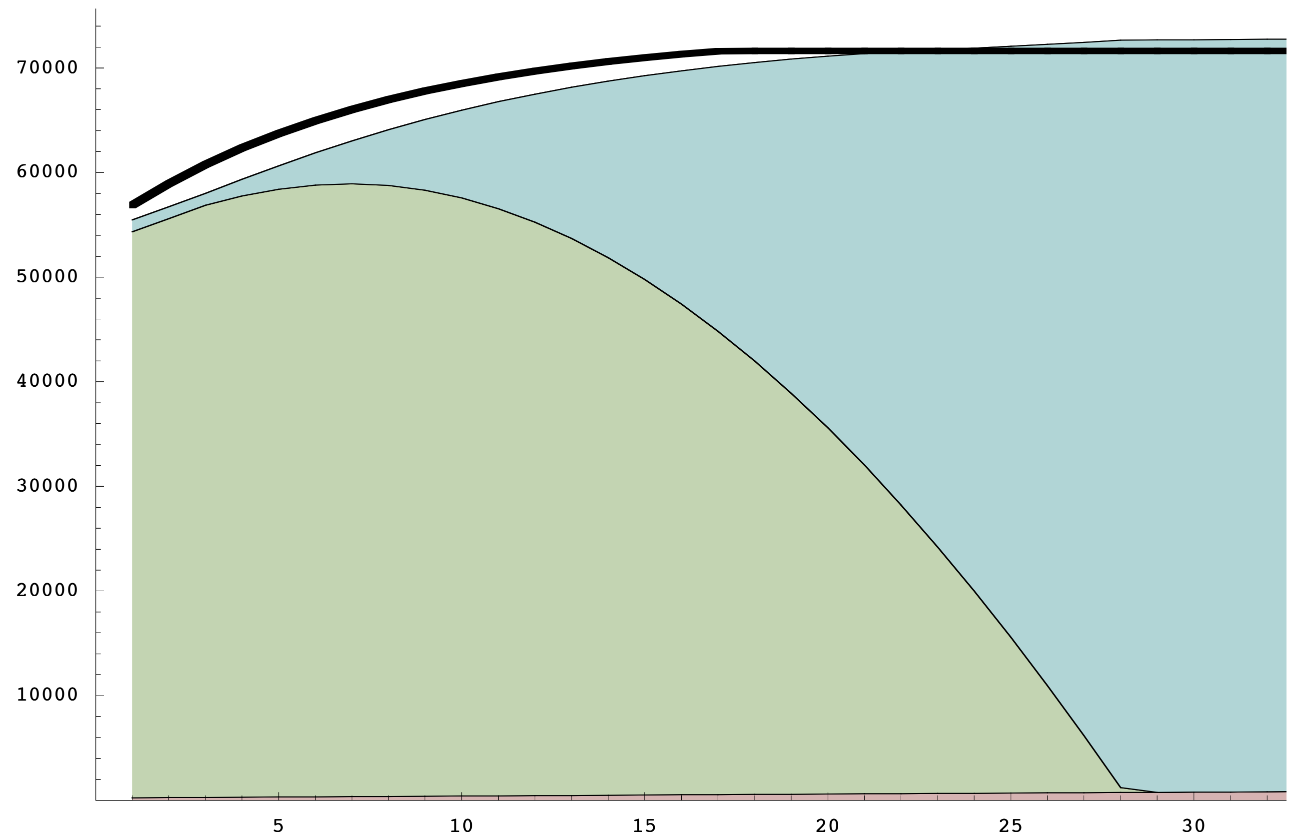
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	1
Best	keep	keep	keep	keep	keep	keep	keep	keep	keep	keep	keep	keep	keep	keep	keep	keep	k
Second Best	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

```
In[99]:=  replNoResale2 = Plus @@ (replJunk /. {V_i_ :> ViSolved2[[i]]});
          replBestYear2 = Ordering[replNoResale2, 1] // First
          replNoResale2[[replBestYear2]]
```

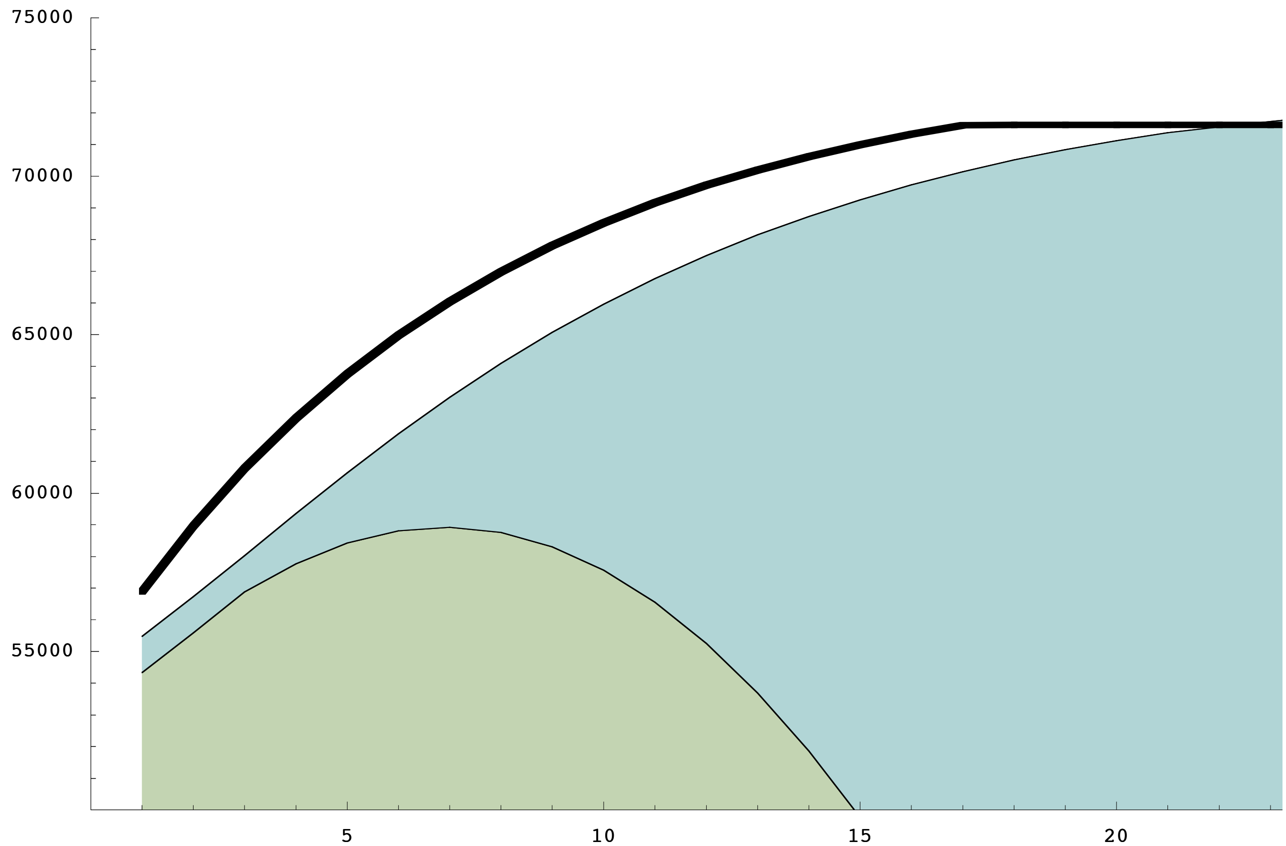
```
Out[100]=
          1
```

```
Out[101]=
          72024.4
```

```
In[102]:=
        DisplayTogether[{
          StackedPlot[keepJunk /. {V_i_ :> ViSolved2[[i]]}],
          ListPlot[Table[replNoResale2[[replBestYear2]] - accordSalePrice[i], {i, 1,
             Last[Dimensions[keepJunk]]}], PlotJoined → True, PlotStyle → Thickness[0.005]]
         }, PlotRange → {{0, 35}, {0, Automatic}} ];
```



In[103]:=
```
DisplayTogether[{
  StackedPlot[keepJunk /. {V_i_ :→ ViSolved2[[i]]}],
  ListPlot[Table[replNoResale2[[replBestYear2]] - accordSalePrice[i], {i, 1,
     Last[Dimensions[keepJunk]]}], PlotJoined → True, PlotStyle → Thickness[0.005]]
 }, PlotRange → {{0, 25}, {50000, 75000}}];
```



Full Model

■ Purchasing Freedom

Initially, we only allowed a car to be replaced with the newest version of the car. In this version of the model, we found the the optimal strategy was to hold onto a car until it completely broke and only then purchase a new car. To add more depth and realism to the model, we allowed transitions into any available used car.

■ Policy Matrix

The eventual outcome of this model is a policy matrix. Given a condition, the policy matrix details what action is optimal. For example, you may want to replace a 2 year old car with a 6 year old model and invest the remainder, rather than experience the loss associated with the depreciation of the younger car.

▪ Honda

In[128]:=

```
(*carvalueModel[aMax,δ,repl,Cbuy,Csell,Ctrans,Cmaint,Pdead]*)
Module[{aMax = 32, nIterations = 100, δ = 1/(1 + 0.03), repl},

  repl = Table[i, {i, 1, 25}];
  {δx4, fi4, Px4, replJunk, keepJunk} = carvalueModel[aMax, δ, repl,
    accordPurchasePrice, accordSalePrice, 2 * Vdaily,
    (BasicMaintenanceCosts[#] + MaintenanceAnnoyanceCosts[#] + 0 #^2) &,
    deathRateHonda];
  ViSolvedAll4 = NestList[Vnext[#, {δx4, fi4, Px4}] &, Array[0 &, aMax + 1], nIterations];
  ViSolved4 = Last[ViSolvedAll4];
  Vnext4 = Vchoices[ViSolved4, {δx4, fi4, Px4}];
  policies4 = Append[repl, "keep"];
 ];

prettyTable[{"Value"}, Range[Length[ViSolved4]], {ViSolved4}]
prettyTable[policies4, Range[Length[ViSolved4]], Vnext4];
prettyTable[{"Best", "Second Best"},
 Range[Length[ViSolved4]], policy[policies4, Vnext4, 2]]
```

Out[129]=

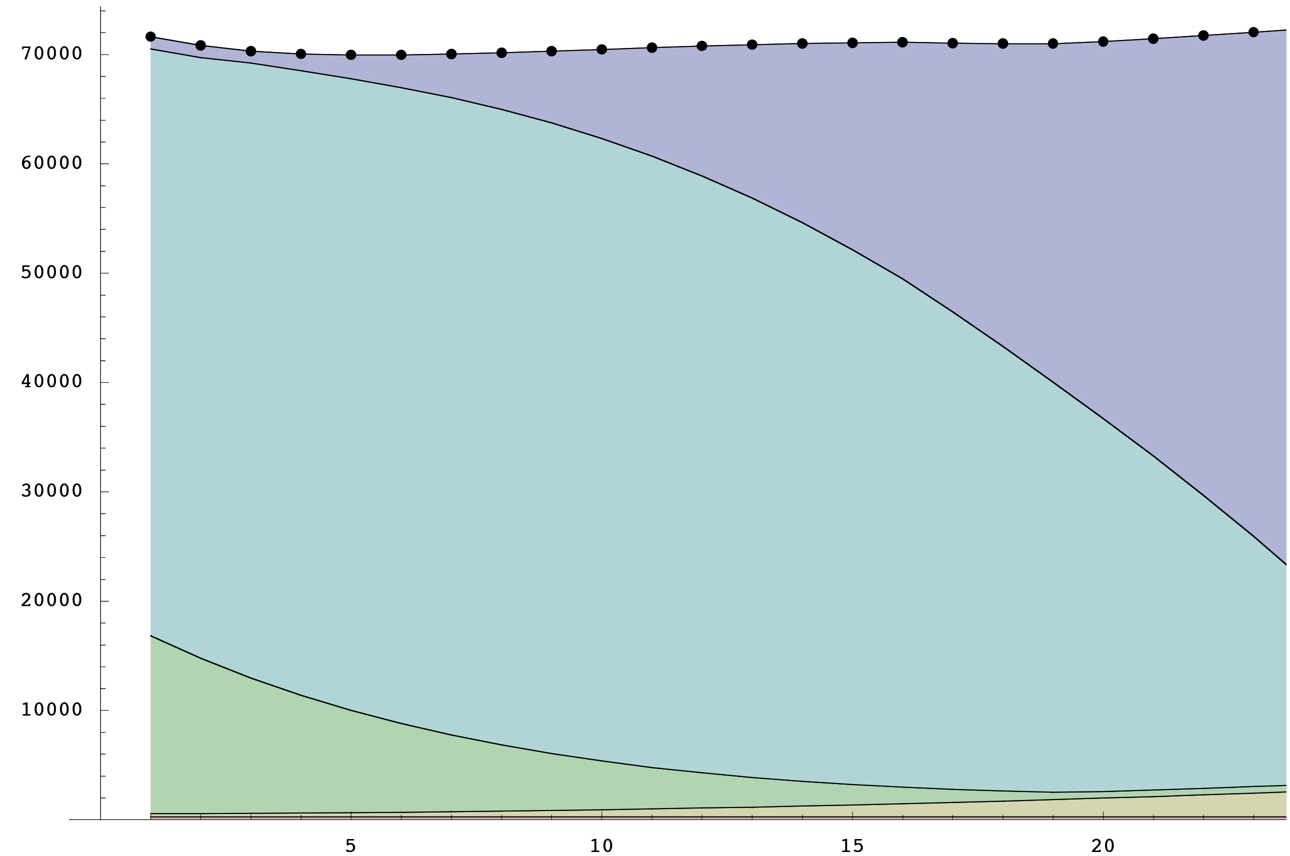
	1	2	3	4	5	6	7	8	9	10
Value	−54695.8	−56202.9	−57505.9	−58850.4	−60158.1	−61406.3	−62579.4	−63668.2	−64667.6	−65576.[illegible]

Out[131]=

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Best	5	keep	keep	keep	keep	keep	keep	keep	keep	keep	keep	keep	keep	keep	keep	5	5
Second Best	6	5	5	5	5	5	5	5	5	5	5	5	5	5	5	6	keep

In[105]:=

```
replNoResale4 = Plus @@ (replJunk /. {Vi_ :> ViSolved4[[i]]});
DisplayTogether[{
   StackedPlot[replJunk /. {Vi_ :> ViSolved4[[i]]}],
   ListPlot[replNoResale4]
  }];
replBestYear4 = Ordering[replNoResale4, 1] // First
replNoResale4[[replBestYear4]]
```



Out[107]=

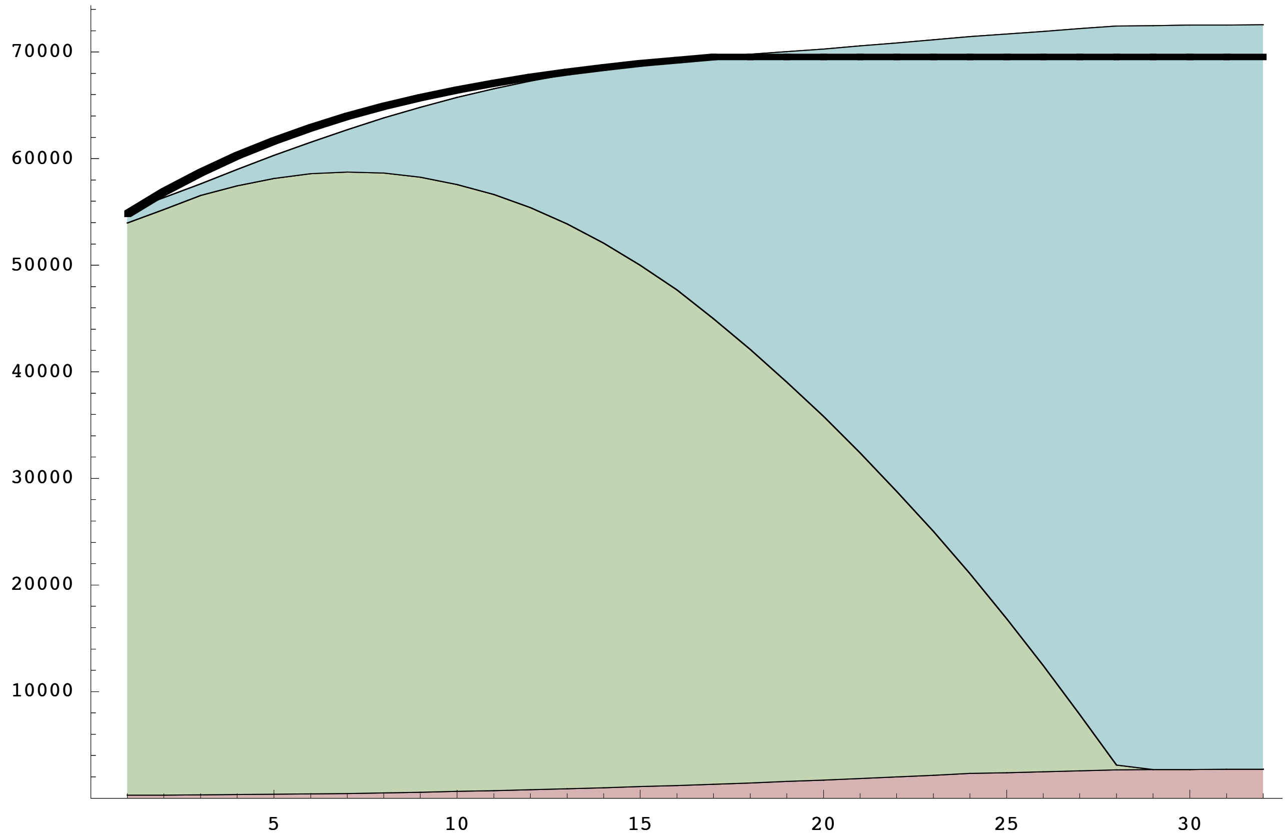
5

Out[108]=

69955.5

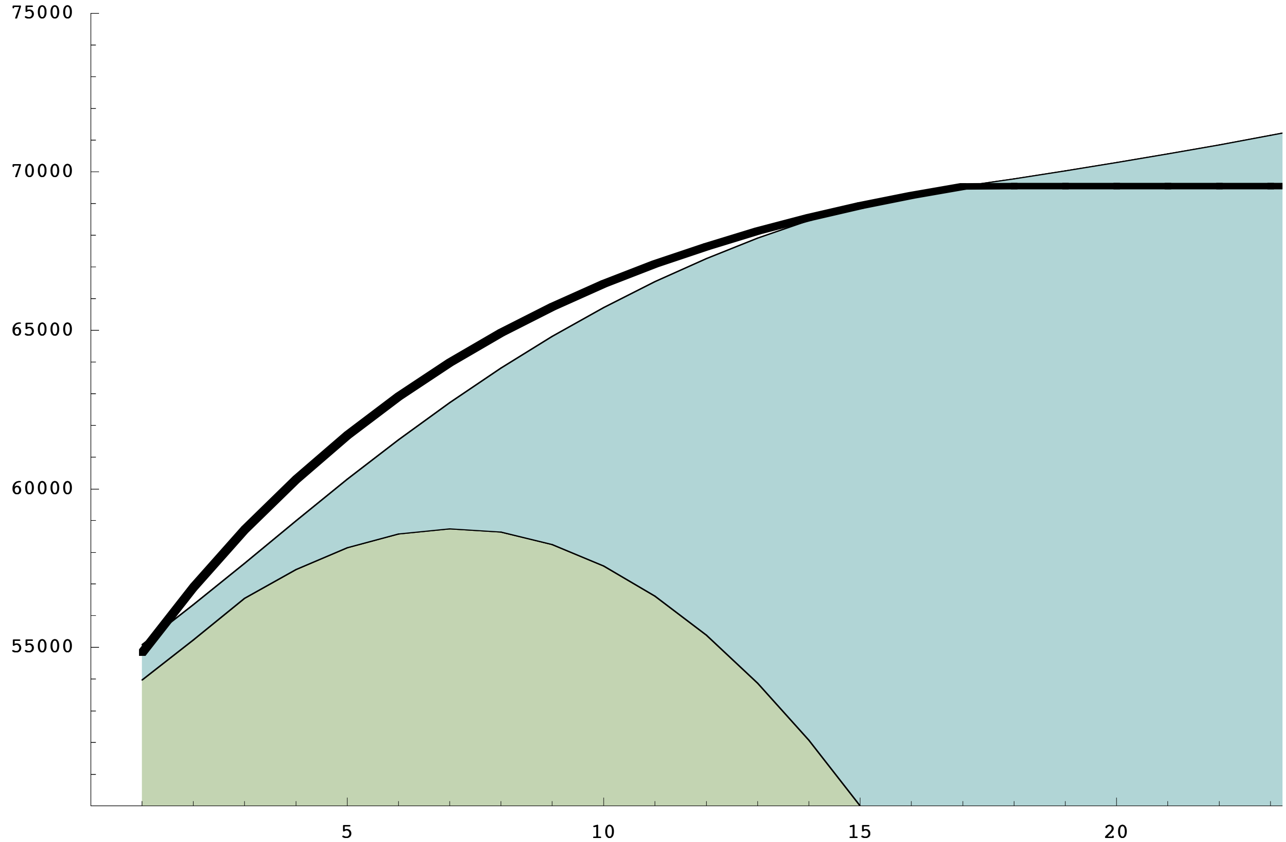
In[109]:=

```
DisplayTogether[{
  StackedPlot[keepJunk /. {Vi_ :→ ViSolved4[[i]]}],
  ListPlot[Table[replNoResale4[[replBestYear4]] - accordSalePrice[i], {i, 1,
     Last[Dimensions[keepJunk]]}], PlotJoined → True, PlotStyle → Thickness[0.005]]
 }, PlotRange → {{0, 35}, {0, Automatic}} ];
```



In[110]:=

```
DisplayTogether[{
  StackedPlot[keepJunk /. {V_i_ :→ ViSolved4[[i]]}],
  ListPlot[Table[replNoResale4[[replBestYear4]] - accordSalePrice[i], {i, 1,
     Last[Dimensions[keepJunk]]}], PlotJoined → True, PlotStyle → Thickness[0.005]]
 }, PlotRange → {{0, 25}, {50000, 75000}}];
```



■ Ford

In[111]:=

```
(*carvalueModel[aMax,δ,repl,Cbuy,Csell,Ctrans,Cmaint,Pdead]*)
Module[{aMax = 34, nIterations = 100, δ = 1/(1 + 0.03), repl},

  repl = Table[i, {i, 1, 25}];
  {δx4F, fi4F, Px4F, replJunk, keepJunk} = carvalueModel[aMax, δ, repl,
    crownvicPurchasePrice, crownvicSalePrice, 2 * Vdaily,
    (BasicMaintenanceCosts[#] + MaintenanceAnnoyanceCosts[#] + 0 #^2) &,
    deathRateFord];
  ViSolvedAll4F =
   NestList[Vnext[#, {δx4F, fi4F, Px4F}] &, Array[0 &, aMax + 1], nIterations];
  ViSolved4F = Last[ViSolvedAll4F];
  Vnext4F = Vchoices[ViSolved4F, {δx4F, fi4F, Px4F}];
  policies4F = Append[repl, "keep"];
 ];
 prettyTable[{"Value"}, Range[Length[ViSolved4F]], {ViSolved4F}]
 prettyTable[policies4F, Range[Length[ViSolved4F]], Vnext4F];
 prettyTable[{"Best", "Second Best"},
  Range[Length[ViSolved4F]], policy[policies4F, Vnext4F, 2]]
```

General::spell1 : Possible spelling error: new symbol name "δx4F" is similar to existing symbol "δx4". More…

General::spell1 : Possible spelling error: new symbol name "fi4F" is similar to existing symbol "fi4". More…

General::spell1 : Possible spelling error: new symbol name "Px4F" is similar to existing symbol "Px4". More…

General::stop : Further output of General::spell1 will be suppressed during this calculation. More…

Thread::tdlen : Objects of unequal length in

Null

	1	2	3	4	5	6	7	8	9	≪26≫
Value	−40365.6	−42953.6	−44855.4	−45891.	−46941.9	−47966.1	−48933.7	−49825.2	−≪18≫	≪26≫

Null

	1	2	3	4	5	6	7	8	9	≪26≫
Best	7	7	keep	keep	keep	keep	keep	keep	keep	≪26≫
Second Best	6	6	7	7	7	7	7	7	7	≪26≫

cannot be combined. More…

Thread::tdlen : Objects of unequal length in

$Null^2$

	1	2	3	4	5	6	7	8	9	≪26≫
Value	−40365.6	−42953.6	−44855.4	−45891.	−46941.9	−47966.1	−48933.7	−49825.2	−50629.6	≪1≫

	1	2	3	4	5	6	7	8	9	≪26≫
Best	7	7	keep	keep	keep	keep	keep	keep	keep	≪26≫
Second Best	6	6	7	7	7	7	7	7	7	≪26≫

cannot be combined. More…

Out[111]=

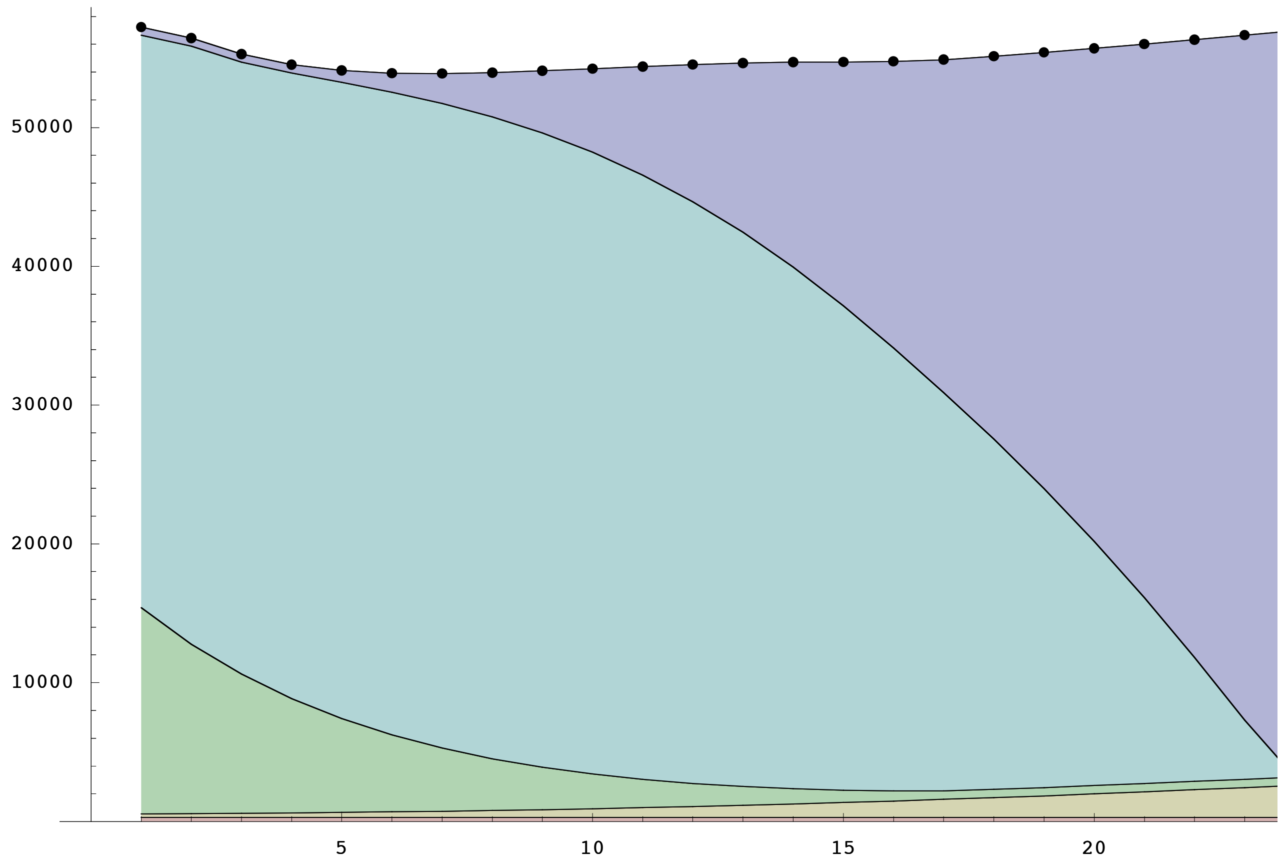
$Null^2$

	1	2	3	4	5	6	7	8	9	10
Value	−40365.6	−42953.6	−44855.4	−45891.	−46941.9	−47966.1	−48933.7	−49825.2	−50629.6	−51:

In[112]:=

```
replNoResale4F = Plus @@ (replJunk /. {V_i_ :→ ViSolved4F[[i]]});
DisplayTogether[{
   StackedPlot[replJunk /. {V_i_ :→ ViSolved4F[[i]]}],
   ListPlot[replNoResale4F]
  }];
replBestYear4F = Ordering[replNoResale4F, 1] // First
replNoResale4F[[replBestYear4F]]
```

General::spell1 : Possible spelling error: new symbol name "replNoResale4F" is similar to existing symbol "replNoResale4". More…



General::spell1 : Possible spelling error: new symbol name "replBestYear4F" is similar to existing symbol "replBestYear4". More…

Out[114]=

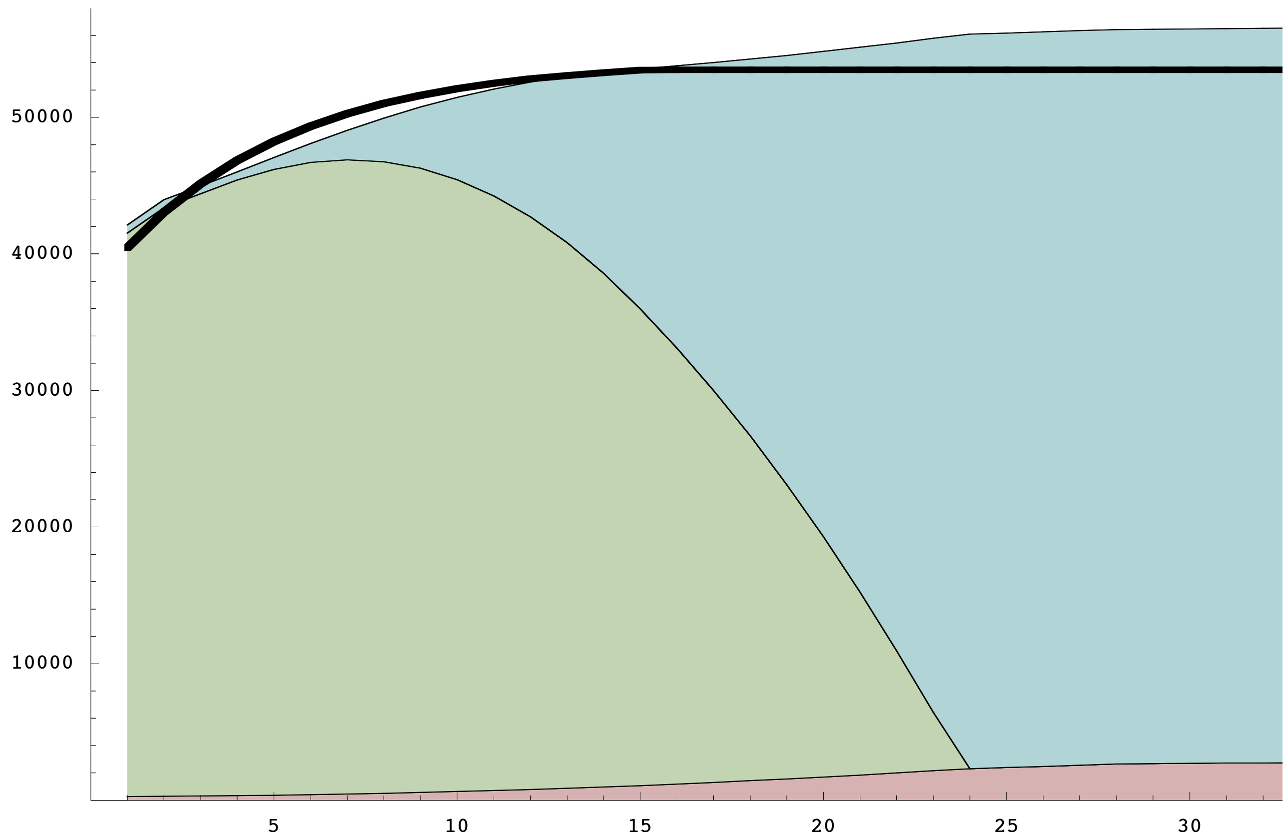
7

Out[115]=

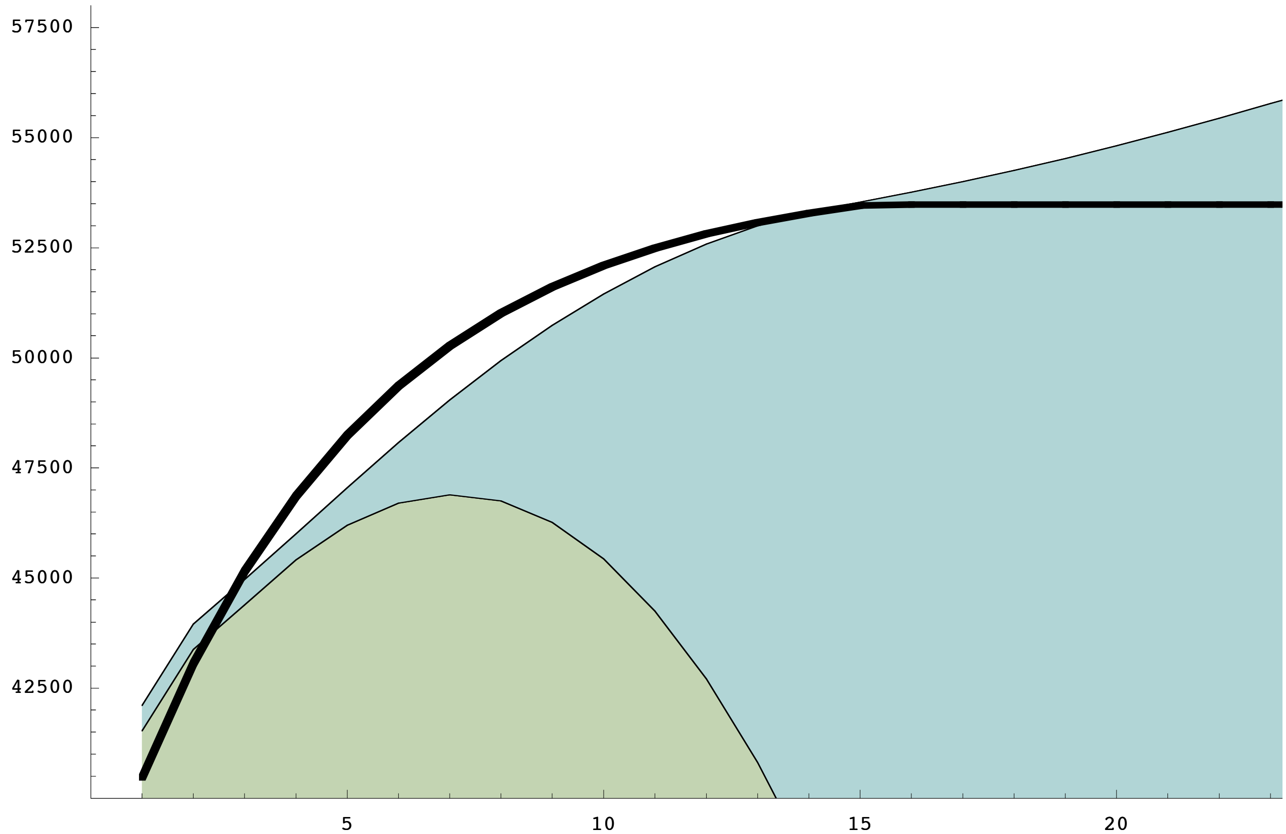
53884.2

In[116]:=

```
DisplayTogether[{
  StackedPlot[keepJunk /. {V_i_ :→ ViSolved4F[[i]]}],
  ListPlot[Table[replNoResale4F[[replBestYear4F]] - crownvicSalePrice[i], {i, 1,
     Last[Dimensions[keepJunk]]}], PlotJoined → True, PlotStyle → Thickness[0.005]]
 }, PlotRange → {{0, 35}, {0, Automatic}} ];
```



In[117]:=
```
DisplayTogether[{
  StackedPlot[keepJunk /. {Vi_ :→ ViSolved4F[[i]]}],
  ListPlot[Table[replNoResale4F[[replBestYear4F]] - crownvicSalePrice[i], {i, 1,
     Last[Dimensions[keepJunk]]}], PlotJoined → True, PlotStyle → Thickness[0.005]]
 }, PlotRange → {{0, 25}, {40000, 58000}}];
```



The Price of that "New Car" Smell

■ Price of New Car Smell?

For a given set of model parameters, it is possible to calculate the extended value function for a car of any age. This value tell you the expected cost of owning the car along with the costs associated with all the cars that will replace it. If you don't have a car, you should purchase the car that has the largest value of the extended difference minus the cost of the car. This choice will then have the cheapest long term ownership cost.

In[118]:=
```
(* Car Maintenance Costs per year by age *)
BasicMaintenanceCosts[a_] := 229.7 + 18.739 a;

(* Market Price for a car *)
accordSalePrice      [age_] =
  (-1626.0533895036542` + 19102.56479673649` e^(-0.131837339952695` age)) // Max[#, 400] &;
accordPurchasePrice  [age_] =
  (-1091.3987901373991` + 19698.233221372142` e^(-0.12658551504176688` age)) // Max[#, 600] &;
crownvicSalePrice    [age_] =
  (-312.1598262326867` + 16910.96076671705` e^(-0.2090073507059101` age)) // Max[#, 400] &;
crownvicPurchasePrice[age_] =
  (-83.0445720243316` + 18138.52244949044` e^(-0.1948081963984956` age)) // Max[#, 600] &;
BasicMarketPrice[age_] = (accordSalePrice[age] + accordPurchasePrice[age]) / 2;
```

In[124]:=
```
marketprice = Table[BasicMarketPrice[i], {i, 1, Length[ViSolved4] - 1}]
```

Out[124]=

{15690.9, 13625., 11809.6, 10214.2, 8812.11, 7579.98, 6497.17, 5545.58, 4709.3, 3974.34, 3328.44, 2760.79, 2261.92, 1823.48, 1438.15, 1099.49, 801.861, 663.182, 543.225, 500, 500, 500, 500, 500, 500, 500, 500, 500, 500, 500, 500, 500}

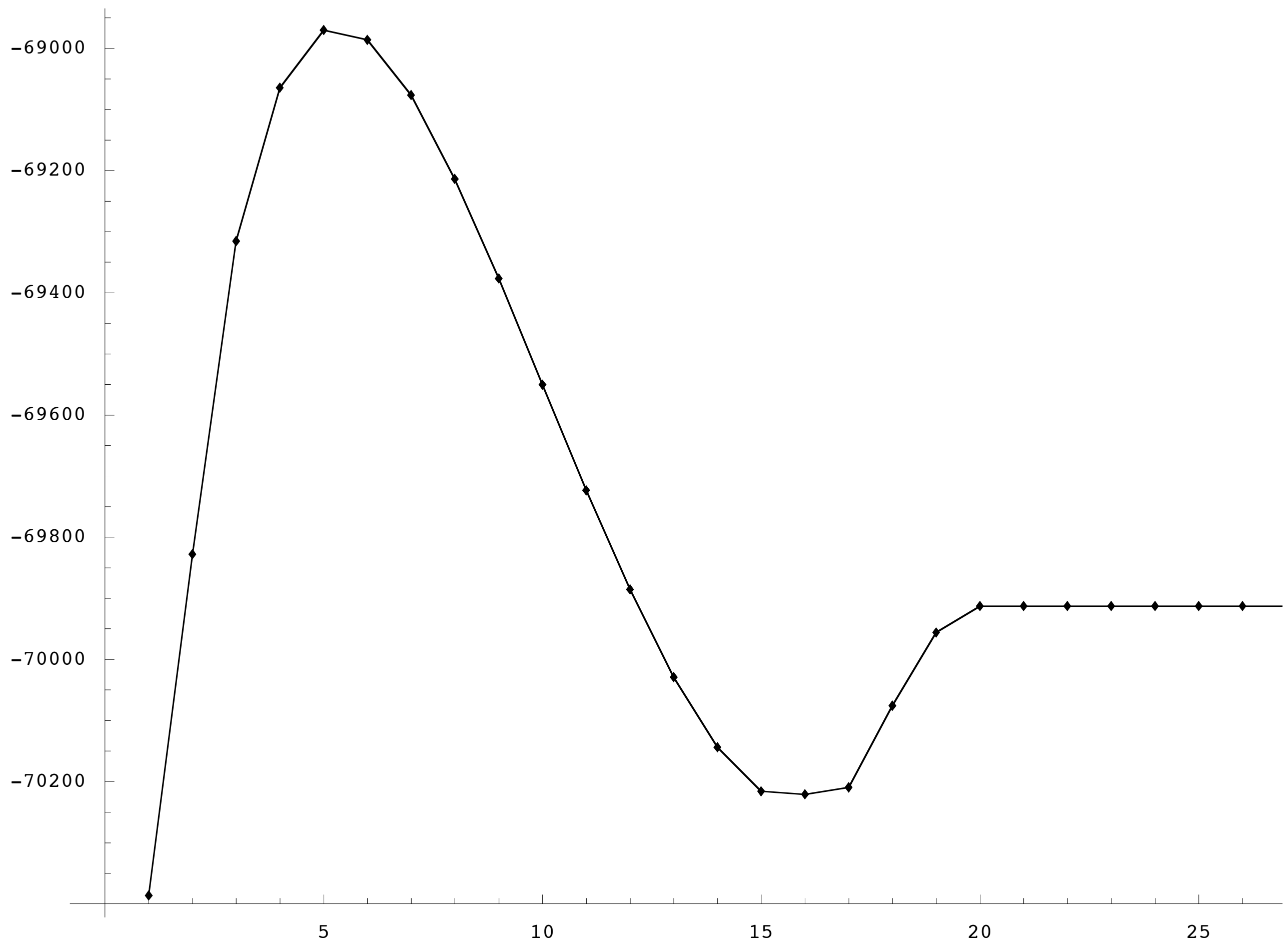
In[125]:=
```
difference = (ViSolved4 // Most) - marketprice
```

Out[125]=

{−70386.7, −69827.9, −69315.5, −69064.5, −68970.2, −68986.3, −69076.6, −69213.7, −69376.9, −69550.6, −69723.4, −69885.8, −70029.3, −70144.2, −70216.2, −70221.1, −70209.7, −70076.1, −69956.1, −69912.9, −69912.9, −69912.9, −69912.9, −69912.9, −69912.9, −69912.9, −69912.9, −69912.9, −69912.9, −69912.9, −69912.9, −69912.9}

In[126]:=

```
MultipleListPlot[difference, PlotJoined → True, ImageSize → {10 * 72, Automatic}];
```

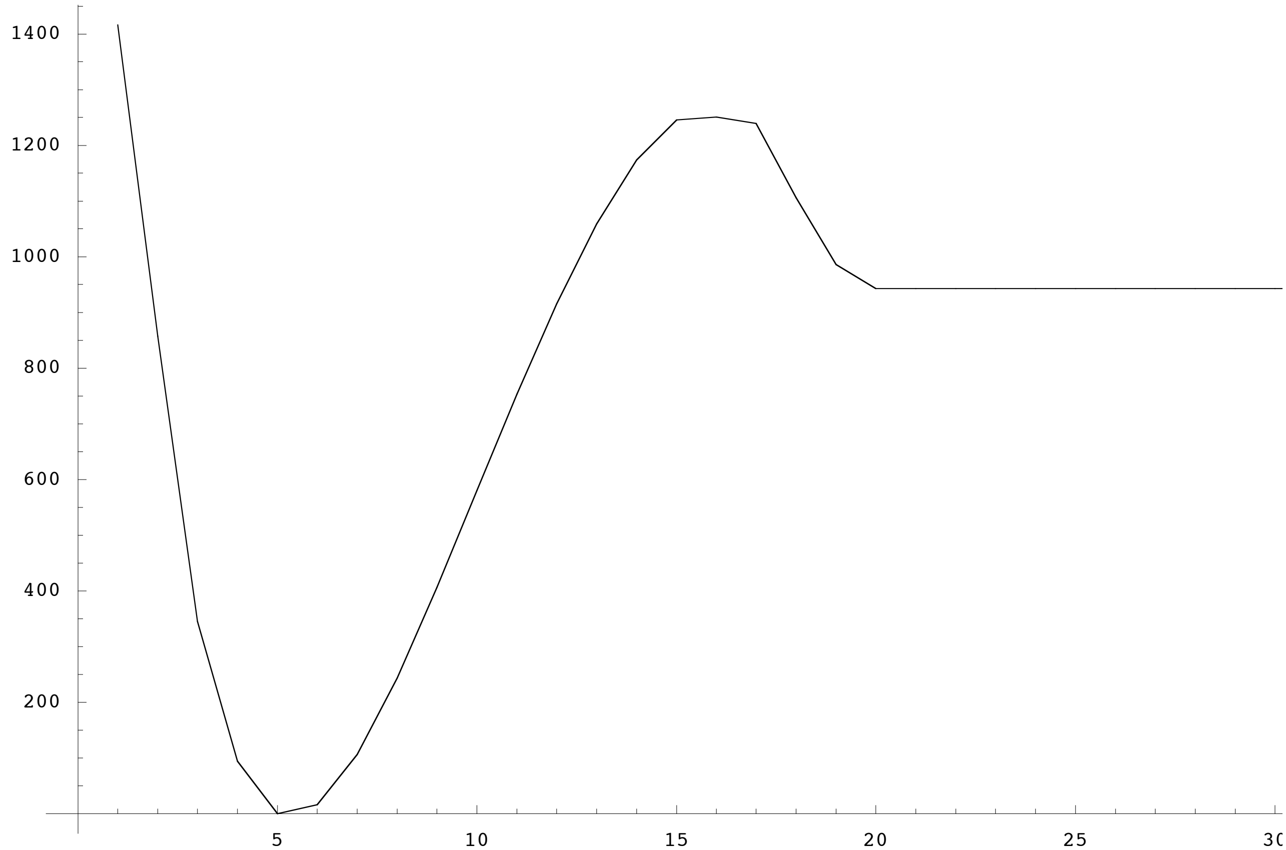


Used cars have a peak value (keep in mind, peak means "least negative") at 5 years old. Yet, cars sell at all all prices. Why is that?

There is more, however, to the value of a car.
⇒ Rather than existing as perfect substitutes, a new car has better features than an old car: current model year cars tend to come with traction control and MP3 players; cars that are more than a few years old have air bags, CD players, and ABS; cars that are more than a decade old will not only lack such ameliorations but may go on the market with even their standard features in disrepair.
⇒ There is also the pure appeal of having a new car -- peace of mind and pride of ownership
⇒ Old cars appreciate in value as they become scarce (see the scrap rate graph)
⇒ Some cars have utility beyond basic transportation -- as a hobby, or for camping, or for attracting a mate

The following graph -- showing the premium people pay above the optimal (age 5) value for a car -- can be said to measure the price of "new-car" smell.

In[127]:=
```
ListPlot[difference[[5]] - difference, PlotJoined → True, ImageSize → {9 * 72, Automatic}];
```



Bibliography

Akerlof, George A., The Market for "Lemons":Quality Uncertainty and the Market Mechanism.
The Quarterly Journal of Economics., The MIT Press., 1970.
Judd, Kenneth L., Numerical Methods in Economics., The MIT Press., 2002
Miranda, Mario J. and Fackler, Paul L., Applied Computational Economics and Finance., The MIT Press., 2002
"Are Lemons Really Hot Potatoes?," Stern, Hartmann and Engers
"Automobile Maintenance Costs, Used Cars, and Adverse Selection," Stern, Hartmann and Engers
"Mileage Drives Used Car Prices," Stern, Hartmann and Engers
http://www.people.virginia.edu/~sns5r/resint/empiostf/empio.html
Graph of Death Rates for Imports and for Domestics taken from S Stern's website:
Domestic http://www.people.virginia.edu/~sns5r/resint/empiostf/selection_giffiles/deathpaths/deathpath2.html
Import http://www.people.virginia.edu/~sns5r/resint/empiostf/selection_giffiles/deathpaths/deathpath3.html